FORMALIZING TYPE THEORY THROUGH TRANSPORT HELL

SZUMI XIE AND VIKTOR BENSE

When formalizing type theory using intrinsically typed syntax in a proof assistant, one encounters "transport hell" [CNT26; KP25]. The transport operation on equality is necessary for terms to be well typed, and normally one needs to define and use a large number of lemmas about the transport operation just to reason about equations of transported terms.

We introduce a technique to deal with transport hell in a metatheory based on observational type theory (OTT) [PT22]. In this technique, we define an interface consisting of a small number of operations on heterogeneous equality, making it easier to work with equations of transported terms. We implemented this technique in the Agda proof assistant by postulating the operations of OTT and adding rewrite rules for computation. Using this technique, we formalized canonicity for a type theory with Π-types, Booleans, and a universe. This is the first formalization of a gluing-style proof of canonicity of type theory which computes terms in a reasonable time, as opposed to the formalization by Kaposi and Pujet [KP25]. The formalization is available at `https://github.com/szumixie/mltt`.

References

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