CCwFs vs. Comprehension Categories

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In this note, I define the notion of *CCwFs*[1], which are groupoid categories with families (GCwFs) [3] that do not require type morphisms to be invertible. At the same time, I compare it to the definition of comprehension categories [4, 1, 2].

1. CONTEXTS AND TYPES

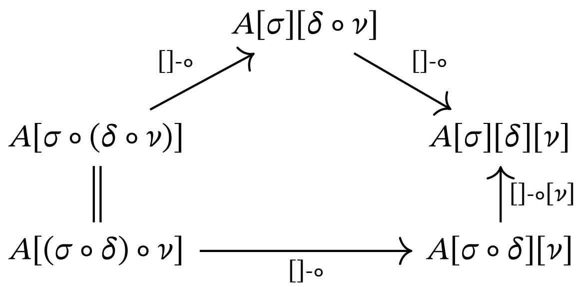
First of all, both CCwFs and comprehension categories start with a category of context and substitutions, which we denote with Con.

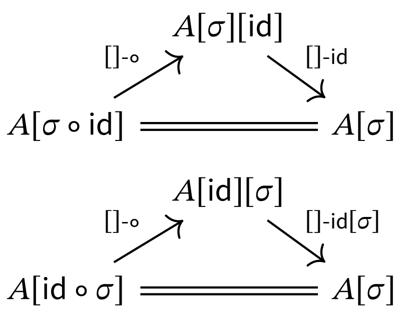
If contexts form a category, then we might say that types form some sort of "dependent category" over contexts. We list three different ways to represent this.

1 *Types form an indexed category over* Con.

DEFINITION 1. An *indexed category* over Con is a pseudofunctor from $\mathsf{Con}^{\mathrm{op}}$ to the bicategory of categories (a pseudopresheaf), it consists of:

- a category $\mathsf{Ty}\ \Gamma$ for any object Γ in Con
- for every morphism $\sigma : \Delta \to \Gamma$ in Con, a functor $-[\sigma]$ from $\mathsf{Ty}\ \Gamma$ to $\mathsf{Ty}\ \Delta$
- isomorphisms $[]\text{-}\circ : A[\sigma \circ \delta] \cong A[\sigma][\delta]$ and $[]\text{-id} : A[\mathsf{id}] \cong A$ natural in A, where A is an object in $\mathsf{Ty}\ \Gamma$
- such that the following diagrams commute:



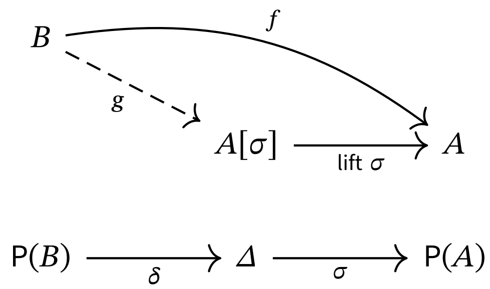


1 *Category CwFs* (?), or maybe it should be *categories with category-indexed families*.

2 *Types form a fibered category* (*Grothendieck fibration*) *over* Con.

DEFINITION 2. A (*cloven*) *fibered category* over Con consists of:

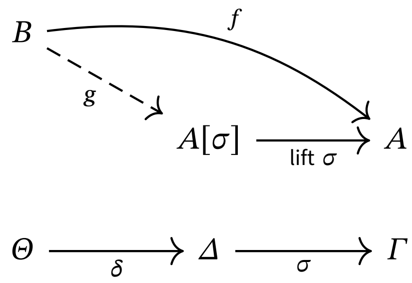
- a category Ty
- a functor P from Ty to Con
- for an object A in Ty and a morphism $\sigma : \Delta \to \mathsf{P}(A)$, we have an object $A[\sigma]$ and a morphism lift $\sigma : A[\sigma] \to A$, such that $\mathsf{P}(A[\sigma]) = \Delta$ and $\mathsf{P}(\text{lift }\sigma) = \sigma$
- such that lift σ is a *cartesian morphism*: for a morphism $f : B \to A$ and a morphism $\delta : \mathsf{P}(B) \to \Delta$ where $\mathsf{P}(f) = \sigma \circ \delta$, there is a unique morphism $g : B \to A[\sigma]$ such that $\mathsf{P}(g) = \delta$ and lift $\sigma \circ g = f$



3 *Types form a fibrant displayed category over* Con.

DEFINITION 3. A *fibrant displayed category* over Con consists of:

- a *displayed category* Ty, which consists of a type of objects Ty Γ for any object Γ in Con and a set of morphisms $f : B \to_\sigma A$ for $\sigma : \Delta \to \Gamma,\ A : \mathsf{Ty}\,\Gamma$, and $B : \mathsf{Ty}\,\Delta$, with operations and equations corresponding to the ones in an ordinary category, but they are "over" the operations and equations of Con
- for an object $A : \mathsf{Ty}\,\Gamma$ and a morphism $\sigma : \Delta \to \Gamma$, we have an object $A[\sigma] : \mathsf{Ty}\,\Delta$ and a morphism lift $\sigma : A[\sigma] \to_\sigma A$
- such that lift σ is a *cartesian morphism*: for a morphism $\delta : \Theta \to \Delta$ and a morphism $f : B \to_{\sigma\circ\delta} A$, there is a unique morphism $g : B \to_\delta A[\sigma]$ such that lift $\sigma \circ g = f$



The three definitions are equivalent through the Grothendieck construction and the displayed–fibered correspondence. CCwFs define types to be an indexed category, while comprehension categories use fibrations (fibered or displayed), but in principle, these could be switched around. We use the displayed-categorical definition of comprehension categories in the rest of this note.

Each of the definitions has a notion of type morphisms, which can be thought of as a subtyping relation on types, or some sort of terms in extended contexts. All of them have a way to substitute a type by a morphism in Con, and the rest of the structures make sure that this substitution is coherent with regards to the type morphisms.

As a lower dimensional example of the correspondence between indexed categories and fibrations, we can consider two ways of defining "dependent setoids" when constructing the setoid model of type theory. Given a setoid $(A, \sim)$, the first definition consists of a type $B\,a$ and a homogeneous equivalence relation $\approx$ on $B\,a$ for any $a : A$, and a coercion operation $\mathsf{coe}\ e : B\,a \to B\,a'$ for an $e : a \sim a'$, such that $\mathsf{coe}\ (\mathsf{trans}\ e\ e')\ b \approx \mathsf{coe}\ e'\ (\mathsf{coe}\ e\ b)$ and $\mathsf{coe}\ \mathsf{refl}\ b \approx b$. The second definition consists of a family B indexed by A, a heterogeneous equivalence relation $\approx$ indexed over $\sim$, and a coercion operation $\mathsf{coe}\ e : B\,a \to B\,a'$ for an $e : a \sim a'$, such that $b \approx_e \mathsf{coe}\ e\ b$.

Another analogy is to consider globular sets with coherences and simplicial/cubical/opetopic sets with Kan filling structure.

2. TERMS AND COMPREHENSION

Like in ordinary CwFs, we have terms in CCwFs. We say that the terms form a covariant presheaf Tm over the covariant Grothendieck construction on the indexed category Ty, which is a generalization of categories of elements.

DEFINITION 4. A *covariant Grothendieck construction* on an indexed category Ty is a category where:

- An object consists of an object Γ in Con and an object A in Ty Γ.
- A morphism from (Γ, B) to (Δ, A) consists of a morphism $\sigma : \Delta \to \Gamma$ in Con and a morphism $f : B[\sigma] \to A$ in Ty Δ.

Note that in the morphisms of this covariant Grothendieck construction, the morphisms in Con and Ty point in opposite directions, this is because Ty is contravariant. This way, a set of terms is indexed over both a context and a type over that context, but the presheaf action on morphisms allows us to substitute terms contravariantly and coerce terms along type morphisms covariantly.

CCwFs also have context comprehension like in CwFs, which is a context extension operation with some universal property. We give the definition of CCwFs below. A completely unfolded definition of CCwFs is listed in appendix A.

DEFINITION 5. A *CCwF* consists of:

- a category Con
- an indexed category Ty
- a covariant presheaf Tm over the covariant Grothendieck construction on Ty
- For an object Γ in Con and an object A in $\mathsf{Ty}\,\Gamma$, we have an object $\Gamma \triangleright A$ in Con, a morphism $\mathsf{p} : \Gamma \triangleright A \to \Gamma$, and an element $\mathsf{q} : \mathsf{Tm}\,(\Gamma \triangleright A)\,A[\mathsf{p}]$, such that for any $\sigma : \Delta \to \Gamma$ and $t : \mathsf{Tm}\,\Delta\,A[\sigma]$, there is a unique morphism $\delta : \Delta \to \Gamma \triangleright A$ such that $\mathsf{p} \circ \delta = \sigma$ and $\mathsf{q}[\delta, []\text{-}\circ^{-1}] = t$

Context comprehension relates terms and substitutions. In fact, terms are isomorphic to sections of p, that is, $\mathsf{Tm}\,\Gamma\,A \cong (\sigma : \mathsf{Sub}\,\Gamma\,(\Gamma \triangleright A)) \times (\mathsf{p} \circ \sigma = \mathsf{id})$. So comprehension categories eschew having a separate set of terms, and the operations on terms are expressed using substitutions, like in categories with attributes (CwAs).

Comprehension categories have a *comprehension functor*, which is a (displayed) functor from the displayed category Ty to the displayed category of slices of Con, giving us the following:

- Action on objects: For an object Γ in Con and an object $A : \mathsf{Ty}\,\Gamma$, we have an object $\Gamma \triangleright A$ in Con and a morphism $\mathsf{p} : \Gamma \triangleright A \to \Gamma$.
- Action on morphisms: For a morphism $\sigma : \Delta \to \Gamma$ and a morphism $f : B \to_\sigma A$, we have a morphism $\sigma \triangleright f : \Delta \triangleright B \to \Gamma \triangleright A$ such that $\mathsf{p} \circ (\sigma \triangleright f) = \sigma \circ \mathsf{p}$.
- The action on morphisms $\triangleright$ preserves composition and identity.

The action on morphisms here is a generalized version of the $\sigma^+ : \Delta \triangleright A[\sigma] \to \Gamma \triangleright A$ operation in CwAs/CwFs. It can embed type morphisms into substitutions in addition to lifting substitutions over the last variable. Then, like in CwAs, we just need a commutative square involving it to form a pullback square, from which we can derive term substitution when terms are defined to be a subset of substitutions. Having this pullback square is equivalent to stating that the comprehension functor preserves cartesian morphisms.

DEFINITION 6. A *comprehension category* consists of:

- a category Con
- a fibrant displayed category Ty
- a functor $(\triangleright, \mathsf{p})$ from Ty to $\mathsf{Con}/-$ (slices of Con) over Con

- such that $(\triangleright, \mathsf{p})$ preserves cartesian morphisms, or equivalently, the following square is a pullback square

$$\begin{array}{ccc} \Delta \triangleright A[\sigma] & \xrightarrow{\sigma \triangleright \mathsf{lift}\ \sigma} & \Gamma \triangleright A \\ {\scriptstyle \mathsf{p}}\downarrow & \lrcorner & \downarrow{\scriptstyle \mathsf{p}} \\ \Delta & \xrightarrow[\sigma]{} & \Gamma \end{array}$$

A completely unfolded definition of comprehension categories is listed in appendix B.

3. COMPARISON

A CCwF consists of a category of contexts, an indexed category of types, and some structure that generalizes the structure in CwFs. A comprehension category consists of a category of contexts, a fibration of types, and some structure that generalizes the structure in CwAs. Since indexed categories and fibrations are equivalent, and CwFs and CwAs are equivalent, we could have the following conjecture:

CONJECTURE 7. *Some bicategory of CCwFs is equivalent to the bicategory of comprehension categories.*

We could also correlate some of the subclasses of CCwFs and comprehension categories:

- A discrete comprehension category (CwA) corresponds to a *discrete CCwF*, which is a CCwF where the category $\mathsf{Ty}\ \Gamma$ is discrete for any Γ, from this we get an ordinary CwF.
- A full comprehension category corresponds to a *full CCwF*, which is a CCwF where the map $f \mapsto \mathsf{q}[\mathsf{id}, f[\mathsf{p}] \circ [] \text{-} \mathsf{id}]$ from $B \to A$ to $\mathsf{Tm}\ (\Gamma \triangleright B)\ A[\mathsf{p}]$ is an isomorphism. Both non-full comprehension categories and CCwFs have type morphisms, which do not comprise all terms in extended contexts, this is what makes it possible to prove that types in the GCwF syntax of type theory form a set [3]. Fullness identifies type morphisms and terms in extended contexts.
- A *GCwF* is a CCwF where the category $\mathsf{Ty}\ \Gamma$ is a groupoid for any Γ, this corresponds to a comprehension category where the fibration of types is fibered in groupoids, that is, every fiber of Ty is a groupoid. The GCwFs presented by Altenkirch, Kaposi, and Xie [3] are GCwFs where Ty is a pseudofunctor from $\mathsf{Con}^{\mathrm{op}}$ to the bicategory of h-groupoids, which are equivalent to “type-univalent” GCwFs, the description of these GCwFs are shorter, as one can work with h-groupoids synthetically.

We list the components of a completely unfolded definition of CCwFs below.

$$
\begin{array}{ll}
\mathsf{Con} & : \mathrm{Type} \\
\mathsf{Sub} & : \mathsf{Con} \to \mathsf{Con} \to \mathrm{Set} \\
- \circ - & : \mathsf{Sub}\,\Delta\,\Gamma \to \mathsf{Sub}\,\Theta\,\Delta \to \mathsf{Sub}\,\Theta\,\Gamma \\
\mathsf{assoc} & : \sigma \circ (\delta \circ \nu) = (\sigma \circ \delta) \circ \nu \\
\mathsf{id} & : \mathsf{Sub}\,\Gamma\,\Gamma \\
\mathsf{idr} & : \sigma \circ \mathsf{id} = \sigma \\
\mathsf{idl} & : \mathsf{id} \circ \sigma = \sigma
\end{array}
$$

$$
\begin{array}{ll}
\mathsf{Ty} & : \mathsf{Con} \to \mathrm{Type} \\
\mathsf{Tym} & : \mathsf{Ty}\,\Gamma \to \mathsf{Ty}\,\Gamma \to \mathrm{Set} \\
- \circ - & : \mathsf{Tym}\,B\,A \to \mathsf{Tym}\,C\,B \to \mathsf{Tym}\,C\,A \\
\mathsf{assoc} & : f \circ (g \circ h) = (f \circ g) \circ h \\
\mathsf{id} & : \mathsf{Tym}\,A\,A \\
\mathsf{idr} & : f \circ \mathsf{id} = f \\
\mathsf{idl} & : \mathsf{id} \circ f = f
\end{array}
$$

$$
\begin{array}{ll}
-[-] & : \mathsf{Ty}\,\Gamma \to \mathsf{Sub}\,\Delta\,\Gamma \to \mathsf{Ty}\,\Delta \\
-[-] & : \mathsf{Tym}\,B\,A \to (\sigma : \mathsf{Sub}\,\Delta\,\Gamma) \to \mathsf{Tym}\,B[\sigma]\,A[\sigma] \\
\circ\text{-}[] & : (f \circ g)[\sigma] = f[\sigma] \circ g[\sigma] \\
\mathsf{id}\text{-}[] & : \mathsf{id}[\sigma] = \mathsf{id}
\end{array}
$$

$$
\begin{array}{ll}
[]\text{-}\circ & : (A : \mathsf{Ty}\,\Gamma)\,(\sigma : \mathsf{Sub}\,\Delta\,\Gamma)\,(\delta : \mathsf{Sub}\,\Theta\,\Delta) \to \mathsf{Tym}\,A[\sigma \circ \delta]\,A[\sigma][\delta] \\
[]\text{-}\circ^{-1} & : (A : \mathsf{Ty}\,\Gamma)\,(\sigma : \mathsf{Sub}\,\Delta\,\Gamma)\,(\delta : \mathsf{Sub}\,\Theta\,\Delta) \to \mathsf{Tym}\,A[\sigma][\delta]\,A[\sigma \circ \delta] \\
[]\text{-}\circ\text{-invr} & : ([]\text{-}\circ\,A\,\sigma\,\delta) \circ ([]\text{-}\circ^{-1}\,A\,\sigma\,\delta) = \mathsf{id} \\
[]\text{-}\circ\text{-invl} & : ([]\text{-}\circ^{-1}\,A\,\sigma\,\delta) \circ ([]\text{-}\circ\,A\,\sigma\,\delta) = \mathsf{id} \\
[]\text{-}\circ\text{-nat} & : ([]\text{-}\circ\,A\,\sigma\,\delta) \circ f[\sigma \circ \delta] = f[\sigma][\delta] \circ ([]\text{-}\circ\,B\,\sigma\,\delta) \\
[]\text{-assoc} & : ([]\text{-}\circ\,(A[\sigma])\,\delta\,\nu) \circ ([]\text{-}\circ\,A\,\sigma\,(\delta \circ \nu)) =_{\mathsf{assoc}} \\
 & \quad ([]\text{-}\circ\,A\,\sigma\,\delta)[\nu] \circ ([]\text{-}\circ\,A\,(\sigma \circ \delta)\,\nu)
\end{array}
$$

$$
\begin{array}{ll}
[]\text{-id} & : (A : \mathsf{Ty}\,\Gamma) \to \mathsf{Tym}\,A[\mathsf{id}]\,A \\
[]\text{-id}^{-1} & : (A : \mathsf{Ty}\,\Gamma) \to \mathsf{Tym}\,A\,A[\mathsf{id}]
\end{array}
$$

$$
\begin{array}{ll}
[]\text{-id-invr} & : ([]\text{-id}\,A) \circ ([]\text{-id}^{-1}\,A) = \mathsf{id} \\
[]\text{-id-invl} & : ([]\text{-id}^{-1}\,A) \circ ([]\text{-id}\,A) = \mathsf{id} \\
[]\text{-id-nat} & : ([]\text{-id}\,A) \circ f[\mathsf{id}] = f \circ ([]\text{-id}\,B) \\
[]\text{-idr} & : ([]\text{-id}\,(A[\sigma])) \circ ([]\text{-}\circ\,A\,\sigma\,\mathsf{id}) =_{\mathsf{idr}} \mathsf{id} \\
[]\text{-idl} & : ([]\text{-id}\,A)[\sigma] \circ ([]\text{-}\circ\,A\,\mathsf{id}\,\sigma) =_{\mathsf{idl}} \mathsf{id}
\end{array}
$$

$$
\begin{array}{ll}
\mathsf{Tm} & : (\Gamma : \mathsf{Con}) \to \mathsf{Ty}\,\Gamma \to \mathsf{Set} \\
-[- \mid -] & : \mathsf{Tm}\,\Gamma\,B \to (\sigma : \mathsf{Sub}\,\Delta\,\Gamma) \to \mathsf{Tym}\,B[\sigma]\,A \to \mathsf{Tm}\,\Delta\,A \\
[]\text{-}\circ & : t[\sigma \circ \delta \mid f \circ g[\delta] \circ ([]\text{-}\circ\,A\,\sigma\,\delta)] = t[\sigma \mid g][\delta \mid f] \\
[]\text{-id} & : t[\mathsf{id} \mid []\text{-id}\,A] = t
\end{array}
$$

$$
\begin{array}{ll}
- \rhd - & : (\Gamma : \mathsf{Con}) \to \mathsf{Ty}\,\Gamma \to \mathsf{Con} \\
\mathsf{p} & : \mathsf{Sub}\,(\Gamma \rhd A)\,\Gamma \\
\mathsf{q} & : \mathsf{Tm}\,(\Gamma \rhd A)\,A[\mathsf{p}] \\
-,- & : (\sigma : \mathsf{Sub}\,\Delta\,\Gamma) \to \mathsf{Tm}\,\Delta\,A[\sigma] \to \mathsf{Sub}\,\Delta\,(\Gamma \rhd A) \\
\rhd\text{-}\beta_1 & : \mathsf{p} \circ (\sigma, t) = \sigma \\
\rhd\text{-}\beta_2 & : \mathsf{q}[\sigma, t \mid []\text{-}\circ^{-1}\,A\,\mathsf{p}\,(\sigma, t)] =_{\rhd\text{-}\beta_1} t \\
\rhd\text{-}\eta & : (\mathsf{p} \circ \sigma, \mathsf{q}[\sigma \mid []\text{-}\circ^{-1}\,A\,\mathsf{p}\,\sigma]) = \sigma
\end{array}
$$

B. The GAT of Comprehension Categories

We list the components of a completely unfolded definition of comprehension categories below.

$$
\begin{array}{ll}
\mathsf{Con} & : \mathrm{Type} \\
\mathsf{Sub} & : \mathsf{Con} \to \mathsf{Con} \to \mathsf{Set} \\
- \circ - & : \mathsf{Sub}\,\Delta\,\Gamma \to \mathsf{Sub}\,\Theta\,\Delta \to \mathsf{Sub}\,\Theta\,\Gamma \\
\mathsf{assoc} & : \sigma \circ (\delta \circ \nu) = (\sigma \circ \delta) \circ \nu \\
\mathsf{id} & : \mathsf{Sub}\,\Gamma\,\Gamma \\
\mathsf{idr} & : \sigma \circ \mathsf{id} = \sigma \\
\mathsf{idl} & : \mathsf{id} \circ \sigma = \sigma
\end{array}
$$

$$
\begin{array}{ll}
\mathsf{Ty} & : \mathsf{Con} \to \mathrm{Type} \\
\mathsf{Tym} & : \mathsf{Sub}\,\Delta\,\Gamma \to \mathsf{Ty}\,\Delta \to \mathsf{Ty}\,\Gamma \to \mathsf{Set} \\
- \circ - & : \mathsf{Tym}\,\sigma\,B\,A \to \mathsf{Tym}\,\delta\,C\,B \to \mathsf{Tym}\,(\sigma \circ \delta)\,C\,A
\end{array}
$$

$$
\begin{array}{lll}
\mathsf{assoc} & : & f \circ (g \circ h) = (f \circ g) \circ h \\
\mathsf{id} & : & \mathsf{Tym}\ \mathsf{id}\ A\ A \\
\mathsf{idr} & : & f \circ \mathsf{id} = f \\
\mathsf{idl} & : & \mathsf{id} \circ f = f \\
\\
-[-] & : & \mathsf{Ty}\ \Gamma \to \mathsf{Sub}\ \Delta\ \Gamma \to \mathsf{Ty}\ \Delta \\
\mathsf{lift} & : & (\sigma : \mathsf{Sub}\ \Delta\ \Gamma) \to \mathsf{Tym}\ \sigma\ A[\sigma]\ A \\
\mathsf{unlift} & : & (\sigma : \mathsf{Sub}\ \Delta\ \Gamma) \to \mathsf{Tym}\ (\sigma \circ \delta)\ B\ A \to \mathsf{Tym}\ \delta\ B\ A[\sigma] \\
[]\text{-}\beta & : & \mathsf{lift}\ \sigma \circ \mathsf{unlift}\ \sigma\ f = f \\
[]\text{-}\eta & : & \mathsf{unlift}\ \sigma\ (\mathsf{lift}\ \sigma \circ f) = f \\
\\
- \triangleright - & : & (\Gamma : \mathsf{Con}) \to \mathsf{Ty}\ \Gamma \to \mathsf{Con} \\
\mathsf{p} & : & \mathsf{Sub}\ (\Gamma \triangleright A)\ \Gamma \\
- \triangleright - & : & (\sigma : \mathsf{Sub}\ \Delta\ \Gamma) \to \mathsf{Tym}\ \sigma\ B\ A \to \mathsf{Sub}\ (\Delta \triangleright B)\ (\Gamma \triangleright A) \\
\mathsf{p}\text{-}\triangleright & : & \mathsf{p} \circ (\sigma \triangleright f) = \sigma \circ \mathsf{p} \\
\triangleright\text{-}\circ & : & (\sigma \circ \delta \triangleright f \circ g) = (\sigma \triangleright f) \circ (\delta \triangleright g) \\
\triangleright\text{-}\mathsf{id} & : & (\mathsf{id} \triangleright \mathsf{id}) = \mathsf{id} \\
\\
\mathsf{subst} & : & (\sigma : \mathsf{Sub}\ \Delta\ \Gamma)\ (\delta : \mathsf{Sub}\ \Theta\ \Delta)\ (\nu : \mathsf{Sub}\ \Theta\ (\Gamma \triangleright A)) \to \\
& & \mathsf{p} \circ \nu = \sigma \circ \delta \to \mathsf{Sub}\ \Theta\ (\Delta \triangleright A[\sigma]) \\
\mathsf{p}\text{-}\mathsf{subst} & : & (\sigma : \mathsf{Sub}\ \Delta\ \Gamma)\ (\delta : \mathsf{Sub}\ \Theta\ \Delta)\ (\nu : \mathsf{Sub}\ \Theta\ (\Gamma \triangleright A)) \to \\
& & (e : \mathsf{p} \circ \nu = \sigma \circ \delta) \to \mathsf{p} \circ (\mathsf{subst}\ \sigma\ \delta\ \nu\ e) = \delta \\
\mathsf{subst}\text{-}\beta & : & (\sigma : \mathsf{Sub}\ \Delta\ \Gamma)\ (\delta : \mathsf{Sub}\ \Theta\ \Delta)\ (\nu : \mathsf{Sub}\ \Theta\ (\Gamma \triangleright A)) \to \\
& & (e : \mathsf{p} \circ \nu = \sigma \circ \delta) \to (\sigma \triangleright \mathsf{lift}\ \sigma) \circ (\mathsf{subst}\ \sigma\ \delta\ \nu\ e) = \nu \\
\mathsf{subst}\text{-}\eta & : & (\sigma : \mathsf{Sub}\ \Delta\ \Gamma)\ (\delta : \mathsf{Sub}\ \Theta\ (\Delta \triangleright A[\sigma])) \to \\
& & \mathsf{subst}\ \sigma\ (\mathsf{p} \circ \delta)\ ((\sigma \triangleright \mathsf{lift}\ \sigma) \circ \delta)\ (\ldots) = \delta
\end{array}
$$

REFERENCES

[1] Benedikt Ahrens and Peter LeFanu Lumsdaine. "Displayed Categories". In: *Logical Methods in Computer Science* Volume 15, Issue 1, 20 (Mar. 2019). ISSN: 1860-5974. DOI: 10.23638/LMCS-15(1:20)2019.

[2] Benedikt Ahrens, Peter LeFanu Lumsdaine, and Paige Randall North. "Comparing Semantic Frameworks for Dependently-Sorted Algebraic Theories". In: *Programming Languages and Systems*. Ed. by Oleg Kiselyov. Singapore: Springer Nature Singapore, 2025, pp. 3–22. ISBN: 978-981-97-8943-6.

[3] Thorsten Altenkirch, Ambrus Kaposi, and Szumi Xie. *The Groupoid-Syntax of Type Theory is a Set*. 2025. arXiv: 2509.14988 [cs.LO].

[4] Bart Jacobs. "Comprehension categories and the semantics of type dependency". In: *Theoretical Computer Science* 107.2 (1993), pp. 169–207. ISSN: 0304-3975. DOI: 10.1016/0304-3975(93)90169-T.